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THE TRANSECT CENSUS OF DEER IN THE PASTORAL ZONE OF POINT REYES NATIONAL SEASHORE

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SUMMARY

A transect system of census of deer in the pastoral zone of Point Reyes National Seashore failed to adequately determine the numbers of axis and fallow deer there. A total of 1133 ± 459 blacktail deer were estimated by the same system for a density of 24.7 blacktail/mi^2 (0.13 1/ha), a 38% increase since hunting was banned in 1971. The sex, age ratios of blacktail, axis and fallow deer indicate possible stress on the populations due to a reached or exceeded carrying capacity.

INTRODUCTION

Point Reyes National Seashore, located in Marin County, California, has three species of deer living within its boundaries. The two exotic species, fallow deer (Dama dama), and axis deer (Axis axis) were introduced in the late 1940's by Dr. Millard Ottinger on his ranch on Inverness Ridge. Since then their numbers have increased to a point where it is feared habitat degradation and competition with dairy and beef cattle, and the native blacktail deer (Odocoileus hemionus columbianus) for forage will result.

Therefore, there is a need for a valid and repeatable census method in the pastoral zone of the National Seashore to collect data on the three species to be used in the formulation of management policies. The implementation of the transect system established in this study was an attempt to meet this need. However, unforseen problems rendered the study statistically unuseable for the axis and fallow deer. In order to achieve acceptable confidence limits for the estimation of populations, it was necessary that at least 40 sightings of each species be made along the transects (Burnham et al. 1980). This goal was not reached for the two exotic species due to their clumped distribution. Therefore, most of the data contained herein applies to the blacktail deer of the pastoral zone.

STUDY AREA

The portion of Point Reyes National Seashore considered in this census consists of approximately 21,400 acres; essentially all of the pastoral zone (Figure 1). This region consists primarily of large areas converted to grazeable grassland by bulldozing, burning, plowing and seeding in past years. This conversion is maintained by continuous livestock grazing and periodic plowing, burning, and reseeding (Elliott 1981). Brushy conditions persist on slopes and in ravines too steep to bulldoze. Some land has been plowed and planted in cattle fodder crops, but most of the census area is under heavy grazing pressure from domestic stock. Scattered small stands of Bishop pine (Pinus muricata) are found in some areas. Sand dunes are located immediately inland to some beaches.

METHODS

A 24.5' USGS map was divided into a one kilometer grid along the UTM system. Using this grid, points were plotted along roads at intervals of 500 meters and numbered from 1 to 106. These points served as the potential points of origin for the transects. Using a table of random numbers, 30 of these points were selected. Knowledge of approximate deer density indicated this number of transects, one kilometer in length, should have been sufficient to attain the minimum number of sightings (40 for each species) necessary for acceptable confidence limits (Burnham et al. 1980). Arbitrarily, true East and West were chosen as transect directions and were applied randomly to the chosen points.

Using color aerial photographs, topographic maps, and walking the transects using a compass and pedometer, it was determined if the selected transects were passable on foot for a distance of one kilometer. When a transect was found impassable in the direction selected (East or West), the opposite direction was tried. If this direction also proved to be impassable, the transect was discarded. The table of random numbers was then returned to and the next point of origin and transect direction selected. This process continued until 30 transects were established.

The starting point of each transect was marked at the roadside by driving a four-foot length of rebar painted with floresent orange paint at the top. To enhance visibility, flagging was attached to the rebar and nearby fences or bushes. Where possible, the rebar was driven along a fenceline to reduce the chance of the marker conflicting with vehicles or livestock.

To conduct the census, three teams of two people each walked two transects per day for five days (Dec. 8 - Dec. 12, 1980). The transects were walked in the morning, beginning when light was sufficient. Teams were equipped with two pair of binoculars, compass, rangefinder, pedometer, and clipboard with data sheets and maps showing transect locations. When deer were encountered along a transect, the number was recorded and they were classified according to species, age, and sex (buck, doe, fawn) and antler size of bucks (nub=bump to spike in velvet, I = spike, II - IV = increasing size respectively). Fallow deer were classified by color phase (white, black, common, and menil)

(see sample data sheet). Sighting bearing was determined by compass and distance by rangefinder. Distances under 45 meters (under rangefinders minimum distance) were visually estimated. Distance and bearing were necessary in order to trigonometrically determine the sighting's perpendicular distance from the transect (Figure 2). A transect was walked until the pedometer indicated one kilometer had been covered.

When compiling the data, all sightings over 250 meters perpendicular distance from the transect were disregarded to minimize transec overlap. Calculation of estimated populations was according to "Robust Estimation Methods" in Burnham (et al. 1980) (See Appendix 1).

RESULTS

Calculation of blacktail deer transect data according to Burnham (et al. 1980)(Appendix 1) resulted in an estimate of 1133 blacktail ± 458 in the pastoral zone or 0.131 blacktail per hectare. A breakdown of the sightings used in these calculations is presented in Table 1.

As mentioned before in the introduction, the transect system proved unsuccessful in estimating the numbers of axis and fallow deer by falling far short of supplying the necessary number of sightings (Table 2). However, some composition data were collected and are summarized in Table 3.

DISCUSSION

The census of deer in the National Seashore by Elliott and Wehausen (1971)estimated 24.7 blacktail deer per square mile in the pastoral zone. This figure is substantially lower than the 33.9 blacktail per square mile estimated by my study. The fact that the 1971 census was undertaken soon after deer hunting was prohibited in the Seashore suggests that the lower estimate reflects the affect of hunting pressure on the population. The more recent, higher estimate may show the blacktail population's ability to grow in the absence of hunting. However, the 1971 study showed a sex ration of 27 bucks : 100 does, noting that this is normal for a population in which bucks alone are hunted, and which is at or above carrying capacity of the habitat (Longhurst et al., 1952). This is not significantly different than the 28 bucks : 100 does ratio found by my study. This suggests the possibility that without hunting, carrying capacity has been reached or exceeded and somehow has the same determining affect on sex ratio as hunting; that is a higher mortality rate in bucks.

The blacktail ratio of 31 fawns : 100 does in this study is substantially lower than the 56 fawns : 100 does found in 1971. This, too, may show the affect of carrying capacity on the increased population.

This difference may also be partially due to observer experience in my census. At the time of year the census was conducted, distinguishing fawns of the year from does was often by subtle

differences in size and head characteristics. These differences are often difficult to recognize unless one has experience observing deer.

The sex, age ratio of 69 bucks : 100 does : 34 fawns observed in fallow deer in this census is considerably different than the 57 bucks : 100 does : 59 fawns observed in the pastoral zone by Elliott (1977). The higher proportion of bucks found in the present census indicates a possible change in recent years in mortality factors on bucks, increasing their rate of survival. It is possible that in 1977 the fallow population was still feeling the affects of the drought that could have intensified factors that naturally have a greater negative influence on bucks than does.

The 34 fawns : 100 does of fallow deer in this census compared to the 59 fawns : 100 does of 1977 seem to indicate a reduced survival of fawns this year. The reason for this is unclear but among possible answers are increased predation on fawns, or reduced fecundity in a stressed population at or above carrying capacity.

In 1977 Elliott observed a sex, age ratio in axis deer of 50 bucks : 100 does : 54 fawns. These figures compared to the 39 bucks : 100 does : 39 fawns obtained by my census show a similar ratio reduction in bucks and fawns. This also may be due to the influence of carrying capacity on these two groups in the axis deer population.

It must be considered that, for data obtained for axis and fallow deer, a large enough sample of these two species may not have been taken to reflect accurately the true sex and age ratios.

Also, observer experience may have influenced the ratios due to problems in distinguishing fawns from does. This is especially true for axis deer.

COMMENTS on CENSUS METHODS

Despite the efforts of this census, the need still remains for an easily repeatable, statistically valid method of censusing the exotic deer of Point Reyes. Thorough research into past methods used here and elsewhere to census deer is needed to devise a a system meeting these criteria. I am hopeful that the shortcomings discovered in this census provide help in that direction.

In the past nearly all censuses of deer in the National Seashore have been conducted using a method designed by Woody Elliott which relies on thorough repetitive search of sections of land in an effort to locate all deer present. Much valuable data have been obtained by this method. However, some doubt exists whether this method, when used by other persons in subsequent years, can yield comparable data. Also, as Elliott (1977) admits, the untrained observer should expect to spend four full months to complete a census by this method. This amount of time is necessary to gain knowledge of the range topography and deer habits which can only be learned through experience.

Numbers of sightings of axis and fallow deer obtained in the census in this report are much too low to expect that additional transect lines would yield enough sightings to make this census method useable for these two species. However, additional lines may be able to supply additional sightings of blacktail deer to allow narrower confidence limits in subsequent censuses

by this method. Further improvement is possible by thoroughly training observers in recognition of age and sex classes.

LITERATURE CITED

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Table 1: Tally of blacktail deer sighted at less than 250 meters perpendicular distance from transect line; Transect Census, Point Reyes N. S., Dec. 1980.

Total Sighted	Numbers Classified			Ratio		
	Bucks	Does	Fawns	Bucks	Does	Fawns
160	28	101	31	28	100	31

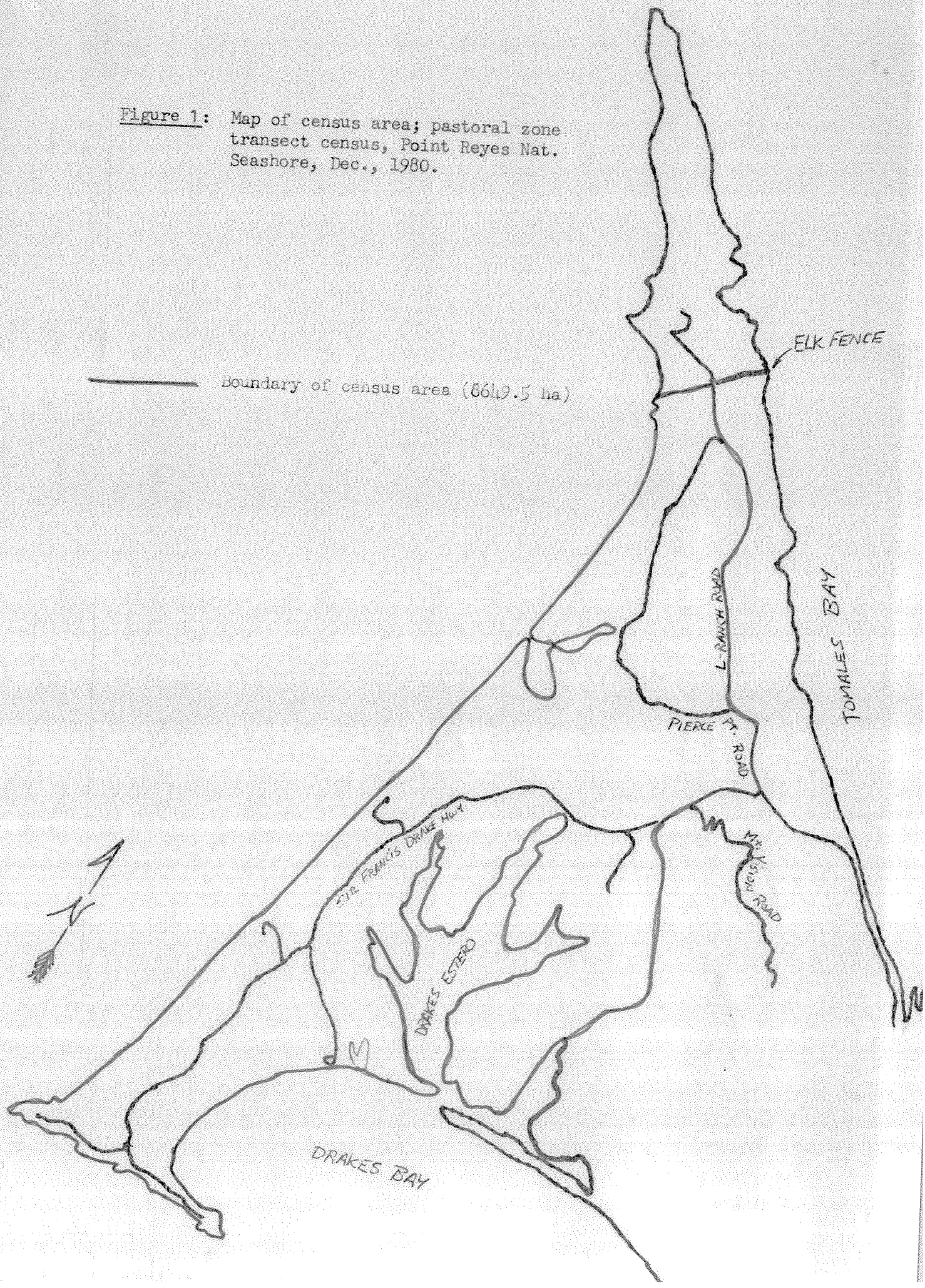
Table 2: Tally of number of sightings; Transect Census, Point Reyes National Seashore, Dec. 1980.

Species	No. Sightings ≤250m	>250m	Totals
Blacktail	42	12	54
Fallow	9	6	15
Axis	4	1	5

Table 3: Tally of deer sighted at all distances from transect line; Transect Census, Point Reyes N. S., Dec. 1980.

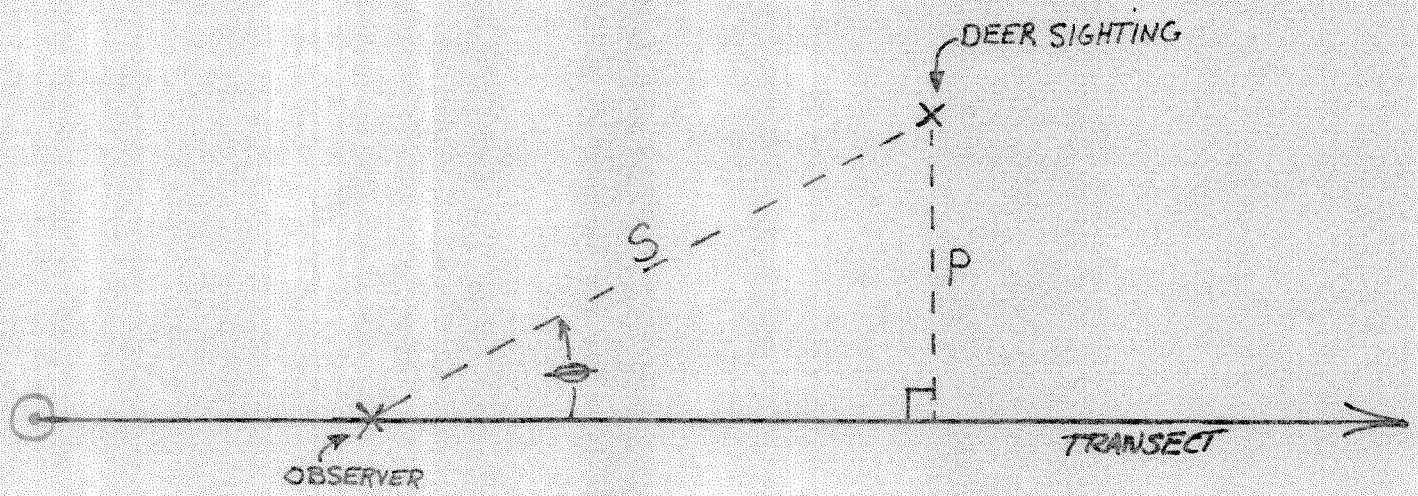
Species	Ratio			Numbers Classified			Total
	Bucks	Does	Fawns	Bucks	Does	Fawns	No. Sighted
Blacktail	24	100	32	30	125	40	195
Fallow	69	100	34	22	32	11	65
Axis	9	100	39	9	23	9	41

Figure 1: Map of census area; pastoral zone transect census, Point Reyes Nat. Seashore, Dec., 1980.



Boundary of census area (6649.5 ha)

FIGURE 2 - DETERMINATION OF SIGHTING'S PERPENDICULAR DISTANCE FROM TRANSECT.



S = SIGHTING DISTANCE (DETERMINED BY RANGEFINDER)

θ = SIGHTING ANGLE (DETERMINED BY COMPASS)

P = PERPENDICULAR DISTANCE OF SIGHTING FROM TRANSECT

$$P = S(\sin\theta)$$

APPENDIX 2

CALCULATIONS

(ESTIMATED BLACKTAIL IN PASTORAL ZONE)

$$\hat{f}(0) = \frac{1}{w} + \sum_{k=1}^{m} \hat{a}_k =$$

$$= \frac{1}{500} + (.002417 + .000513) = 0.00493$$

$$\hat{D} = \frac{n\hat{f}(0)}{2L} = \frac{42 \times 0.00493}{2 \times 30{,}000m} = .00000345 = 3.45 \times 10^{-6} \text{ SIGHTINGS/m}$$

$$= .0345 \text{ SIGHTINGS/ha}$$

$\hat{D}$ x AVG HERD SIZE $= (0.0345 \frac{}{ha})(4.31 \frac{}{DEER}) = 0.149$ DEER/ha

$$\hat{var}(\hat{a}_k) = \frac{1}{n-1}\left[\frac{1}{w}\left(a_{2k} + \frac{2}{w}\right) - a_k^2\right]$$

$$\hat{var}(\hat{a}_2) = \frac{1}{42-1}\left[\frac{1}{500}\left(.00024 + \frac{2}{500}\right) - .000513^2\right] = 8.22 \times 10^{-6}$$

2x2 matrix:

$$\begin{bmatrix} \hat{var}(\hat{a}_1) & cov(\hat{a}_1, \hat{a}_2) \\ cov(\hat{a}_2, \hat{a}_1) & \hat{var}(\hat{a}_2) \end{bmatrix} = \begin{bmatrix} 7.77 \times 10^{-8} & 8.33 \times 10^{-8} \\ 8.33 \times 10^{-8} & 2.00 \times 10^{-7} \end{bmatrix}$$

$$cov(\hat{a}_k, \hat{a}_j) = \frac{1}{n-1}\left[\frac{1}{w}\left(\hat{a}_{k+j} + \hat{a}_{j-k}\right) - (a_k a_j)\right]$$

$$cov(\hat{a}_2, a_1) = \frac{1}{42-1}\left[\frac{1}{500}\left(-.00009 + .002417\right) - (.000513 \times .002417)\right]$$

$$= 8.33 \times 10^{-8}$$

$$\hat{var}(\hat{a}_1) = \frac{1}{42-1}\left[\frac{1}{500}\left(.000513 + \frac{2}{500}\right) - .002417^2\right] = 7.77 \times 10^{-8}$$

$$\hat{var}(\hat{f}(0)) = \sum_{j=1}^{m} \sum_{k=1}^{m} \hat{cov}(a_j, a_k) = \text{SUM OF ALL ELEMENTS OF MATRIX}$$

$$= 7.77 \times 10^{-8} + 2(8.33 \times 10^{-8}) + 2.00 \times 10^{-7} = 4.44 \times 10^{-7}$$

$$\hat{var}(\hat{D}) = \hat{D}^2\left[\frac{1}{n} + \frac{\hat{var}(\hat{f}(0))}{(f(0))^2}\right]$$

$$= (3.45 \times 10^{-6})^2\left[\frac{1}{42} + \frac{4.44 \times 10^{-7}}{(4.93 \times 10^{-3})^2}\right] = 5.01 \times 10^{-13}$$

$$\hat{se}(\hat{D}) = (\hat{var}(\hat{D}))^{\frac{1}{2}} = (5.01 \times 10^{-13})^{\frac{1}{2}} = 7.08 \times 10^{-7}$$

(CONTINUED)

(CONTINUED)

CALCULATIONS (CONTINUED)

AT 95% CONFIDENCE INTERVAL: $\hat{D} \pm 1.96\, \hat{se}(\hat{D}) = \pm 1.39 \times 10^{-6}/m^2$

$= \pm 0.0139/ha$

0.0345 SIGHTINGS/ha ± 0.0139

AVERAGE HERD SIZE $= \frac{160 \text{ DEER}}{42 \text{ SIGHTINGS}} = 3.81$ DEER/SIGHTING

$(3.81 \text{ DEER/SIGHTING})(0.0345 \text{ SIGHTINGS/ha} \pm 0.0139) = 0.131 \pm 0.053$ DEER/ha

$\frac{21,373 \text{ ACRES (TOTAL CENSUS AREA)}}{2.471 \text{ ACRES/ha}} = 8649.5$ ha

$(8649.5 \text{ ha})(0.131 \text{ DEER/ha} \pm 0.053) = 1133 \text{ DEER} \pm 458$

(# OF BLACKTAIL DEER IN TOTAL CENSUS AREA)

PART 2

ROBUST ESTIMATION METHODS

Material in this part concerns the application of the recommended estimation procedures for perpendicular distance data (grouped and ungrouped) and for sighting distance and angle data. PART 2 is directed to the biologist and presents many figures and examples to illustrate the concepts and calculations. In addition, sections cover estimation for mobile populations and grouped data. A final section discusses the use of a comprehensive computer program TRANSECT that should be helpful in the analysis of the line transect data.

The methods described appear to have good properties in comparison with other methods in the literature or with other methods that we developed and examined (cf. Part 4). In PART 2, we stress the application of methods that we believe have good properties. The mathematical theory underlying these estimation methods and computer simulation results on their small sample properties are covered in PART 3.

ANALYSIS AND INFERENCE PROCEDURES FOR PERPENDICULAR DISTANCES, UNGROUPED DATA

Fourier Series Estimator

The estimation procedure described in this section is based on a Fourier series (FS) expansion of the probability density function (pdf), f(x). A Fourier (pronounced Fouréay) series is an expression of sines and cosines and was derived in the 18th century by the French mathematical physicist, Jean Baptiste Joseph Fourier (1768–1830). The Fourier series is one of a number of mathematical expansions of a function. Such series expansions are exact for infinitely many terms. By choosing one such expansion and limiting the number of terms, one arrives at a general model for f(x). An objective rule must then be derived to select the appropriate number of terms for a given set of data. The interest is in approximation with only m terms, where m is a small value, say 6 or less. In general,

$$f(x) \doteq \frac{1}{w^*} + \sum_{k=1}^{m}\left[a_k\cos\left(\frac{k\pi x}{w^*}\right) + b_k\sin\left(\frac{k\pi x}{w^*}\right)\right]$$

Because one is concerned with the estimation of f(0) and sin(0) = 0, the sine terms and the coefficients b_k are not used in the context of line transect analysis.

The Fourier series (FS) is, of course, for periodic and we are concerned with expansions from 0 to w^*, where $w^* \leq$. In this section, only the analysis of ungrouped perpendicular distance data are treated. (Refer to APPENDIX A for a list of symbols and definitions.)

The FS estimator is a nonparametric procedure that is *model robust* and *pooling robust* and meets the *shape criterion* f′(0) = 0. The method is easy to compute and its estimation efficiency for small samples is quite good.

Estimation Equations

The estimation of density from ungrouped data on perpendicular distances is straightforward if the FS approach is used. The procedure employs 7 simple equations. The estimator of density follows from the general theory discussed in PART 1

$$\hat{D} = \frac{n\hat{f}(0)}{2L}. \qquad (2.1)$$

The estimator of the pdf at 0 distance for the FS method is

$$\hat{f}(0) = \frac{1}{w^*} + \sum_{k=1}^{m}\hat{a}_k. \qquad (2.2)$$

Finally, an expression for an estimator of the coefficients a_k is required

$$\hat{a}_k = \frac{2}{nw^*}\left[\sum_{i=1}^{n}\cos\left(\frac{k\pi x_i}{w^*}\right)\right] \quad k = 1, 2, 3, \ldots . \tag{2.3}$$

It is often convenient to denote $2/w^*$ as a_w.

Four additional expressions are required to estimate sampling variances and covariances. The estimator of the sampling variance of the FS coefficients is

$$\hat{v}ar(\hat{a}_k) = \frac{1}{(n-1)}\left[\frac{1}{w^*}\left(\hat{a}_{2k} + \frac{2}{w^*}\right) - \hat{a}_k^2\right] \quad k \geq 1. \tag{2.4}$$

The sampling covariances of those coefficients are estimated from the following expression

$$\hat{c}ov(\hat{a}_k,\hat{a}_j) = \frac{1}{(n-1)}\left[\frac{1}{w^*}(\hat{a}_{k+j} + \hat{a}_{k-j}) - (\hat{a}_k\hat{a}_j)\right] \quad k > j > 1. \tag{2.5}$$

It can be seen from equation (2.2) that $\hat{f}(0)$ is merely the sum of the $\hat{a}_k$ coefficients (m of them) plus a constant $1/w^*$. Therefore, one would expect the sampling variance of $\hat{f}(0)$ to be the sum of all the sampling variances and covariances of the coefficients $\hat{a}_k$ (i.e., equations 2.4 and 2.5). Realizing that $\hat{v}ar(\hat{a}_k)$ can be written as $\hat{c}ov(\hat{a}_k,\hat{a}_k)$, an expression for the sampling variance of $\hat{f}(0)$ is

$$\hat{v}ar(\hat{f}(0)) = \sum_{j=1}^{m}\sum_{k=1}^{m}\hat{c}ov(\hat{a}_j,\hat{a}_k). \tag{2.6}$$

That expression may at first appear formidable, but it is merely the sum of all the elements in the m by m matrix of sampling variances and covariances (from Eq. 2.4 and 2.5). For example, letting m = 3, the following 3 by 3 matrix would be needed:

$$\begin{bmatrix} \hat{v}ar(\hat{a}_1) & \hat{c}ov(\hat{a}_1,\hat{a}_2) & \hat{c}ov(\hat{a}_1,\hat{a}_3) \\ \hat{c}ov(\hat{a}_2,\hat{a}_1) & \hat{v}ar(\hat{a}_2) & \hat{c}ov(\hat{a}_2,\hat{a}_3) \\ \hat{c}ov(\hat{a}_3,\hat{a}_1) & \hat{c}ov(\hat{a}_3,\hat{a}_2) & \hat{v}ar(\hat{a}_3) \end{bmatrix}$$

Again, note that $\hat{v}ar(\hat{a}_k)$ could be expressed as $\hat{c}ov(\hat{a}_k,\hat{a}_k)$. The $\hat{v}ar(\hat{f}(0))$ is merely the sum of all 9 elements in the variance–covariance matrix. Approaches to estimation of var($\hat{D}$) and var(n) are given in PART 1. The use of those equations is illustrated in the examples below; however, first one requires an objective way to choose m, the number of cosine terms in the Fourier series.

Stopping Rule for the Selection of m

The bias in the estimators $\hat{f}(0)$ and $\hat{D}$ can be made arbitrarily small by increasing m, the number of terms in the Fourier series. However, the sampling variance of $\hat{f}(0)$ and $\hat{D}$ increases with increasing values of m. This suggests a trade-off between small bias and large variance. The following stopping rule achieves such a trade-off. Choose the first value of m such that

$$\frac{1}{w^*}\left(\frac{2}{n+1}\right)^{1/2} \geq |\hat{a}_{m+1}| \tag{2.7}$$

where

$|\hat{a}_{m+1}|$ is the absolute value of $\hat{a}_{m+1}$.

The theory for this stopping rule is given in PART 3. That rule appears to be fairly good, and our studies (both theoretical and simulation) provide evidence to suggest that m should not be chosen larger than about 6.

Examples

The use of the FS approach to estimating density is illustrated below with 3 examples. The first 2 are "hand worked" to show the calculations. They were also chosen because the true density is known and, therefore, the bias can be examined. The final example is illustrated by output from the computer program TRANSECT.

Simulated Data

A set of data from our computer simulation studies was chosen to illustrate the use of the Fourier series. Perpendicular distance data x_i were simulated from

TABLE 5.—EXAMPLE OF SIMULATED LINE TRANSECT DATA, AND OTHER STATISTICS, FROM AN UNDERLYING HALF-NORMAL DISTRIBUTION: n = 40, w = 65 m, σ = 33.334, f(0) = 0.0252, L = 1,000, AND D = 0.000504. THE PERPENDICULAR DISTANCES x_i ARE IN METERS

		k = 1		k = 2		k = 3	
i	x_i	$k\pi x_i/w$[1]	$\cos(k\pi x_i/w)$	$k\pi x_i/w$[1]	$\cos(k\pi x_i/w)$	$k\pi x_i/w$[1]	$\cos(k\pi x_i/w)$
1	24.10	1.165	0.395	2.330	−0.688	3.495	−0.938
2	32.92	1.591	−0.020	3.182	−0.999	4.773	0.061
3	8.53	0.412	0.916	0.824	0.679	1.236	0.329
4	36.76	1.777	−0.204	3.554	−0.916	5.330	0.579
5	2.14	0.103	0.995	0.206	0.979	0.309	0.953
6	37.61	1.818	−0.244	3.636	0.880	5.454	0.675
7	0.00	0.000	0.000	0.000	0.000	0.000	1.000
8	43.16	2.086	−0.493	4.172	−0.514	6.258	1.000
9	10.98	0.531	0.862	1.062	0.487	1.593	−0.022
10	43.20	2.088	−0.494	4.176	−0.511	6.264	1.000
11	27.26	1.317	0.251	2.634	−0.874	3.951	−0.690
12	14.27	0.690	0.771	1.380	0.190	2.070	−0.479
13	61.88	2.991	−0.989	5.982	0.955	8.973	−0.900
14	36.33	1.756	−0.184	3.512	−0.932	5.268	0.527
15	5.99	0.290	0.958	0.580	0.836	0.870	0.645
16	16.28	0.787	0.706	1.574	−0.003	2.361	−0.710
17	8.30	0.401	0.921	0.802	0.695	1.203	0.360
18	27.61	1.334	0.234	2.668	−0.890	4.002	−0.652
19	25.25	1.220	0.343	2.440	−0.764	3.660	−0.869
20	16.25	0.785	0.707	1.570	0.001	2.355	−0.706
21	59.69	2.885	−0.967	5.770	0.871	8.655	−0.718
22	25.31	1.223	0.341	2.446	−0.768	3.669	−0.864
23	3.05	0.147	0.989	0.294	0.957	0.441	0.904
24	9.06	0.438	0.906	0.876	0.640	1.314	0.254
25	48.56	2.347	−0.701	4.694	−0.018	7.041	0.726
26	32.68	1.579	−0.009	3.158	−1.000	4.737	0.025
27	23.58	1.140	0.418	2.280	−0.651	3.420	−0.961
28	24.78	1.120	0.365	2.240	−0.620	3.360	−0.976
29	44.42	2.147	−0.545	4.294	−0.406	6.441	0.988
30	6.94	0.335	0.944	0.670	0.784	1.065	0.484
31	13.18	0.637	0.804	1.274	0.292	1.911	−0.334
32	41.01	1.982	−0.400	3.964	−0.680	5.946	0.944
33	48.05	2.322	−0.683	4.644	−0.068	6.966	0.776
34	17.48	0.845	0.664	1.690	−0.119	2.535	−0.822
35	14.99	0.724	0.749	1.448	0.122	2.172	−0.566
36	37.84	1.829	−0.255	3.658	−0.870	5.487	0.699
37	5.32	0.257	0.967	0.514	0.871	0.771	0.717
38	27.73	1.340	0.229	2.680	−0.895	4.020	−0.638
39	23.71	1.146	0.412	2.292	−0.660	3.438	−0.956
n = 40	7.95	0.384	0.927	0.768	0.719	1.152	0.407
			10.586		−2.888		1.332

[1] Values in radians, not degrees.

an underlying truncated half-normal density function

$$f(x) = \frac{\exp\left(-\tfrac{1}{2}\left(\frac{x}{\sigma}\right)^2\right)}{\sqrt{2\pi}\,\sigma\left(\phi\left(\frac{w}{\sigma}\right) - \tfrac{1}{2}\right)},$$

$$0 \leq x \leq w, \qquad 0 < \sigma$$

where $\phi(\cdot)$ is the standard normal cumulative distribution function (cdf).

All the perpendicular distance data x_i were assumed to represent distances in meters. We took the transect width to be 65 m (=w) and generated 40 distances using σ = 33.334. The true density is D = 0.000504 (5.04 objects/ha) and f(0) =

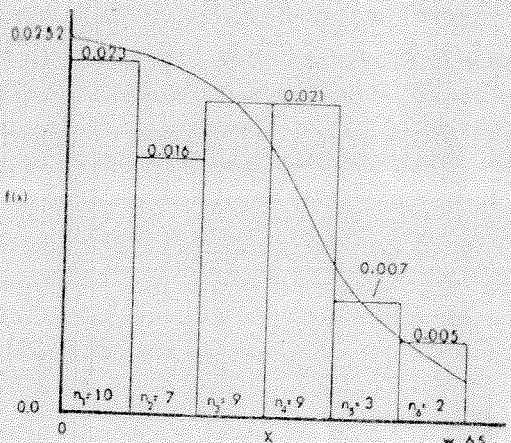


FIG. 26. Truncated half-normal pdf used to illustrate the Fourier series estimator, σ=33.334, w=65 m. Forty perpendicular distances were generated from this distribution. A histogram of the observed frequencies and relative frequencies ($n_i/(\Delta_i n)$) of the data is also shown. Data are grouped into class intervals of 10.83 m.

0.0252. The simulated data are shown in Table 5, with the simple statistics (for k = 1, 2, 3) needed to compute estimates of the FS coefficients a_k. A sample histogram and the underlying pdf are shown in Fig. 26, and Fig. 27 shows the underlying and the estimated pdf.

Thus, n = 40 and w = w* = 65 m (because the data were collected [i.e., simulated] with a fixed width transect, it is convenient to let w* = w) and we assumed L = 1,000 m.

The coefficients a_k are computed by using Eq. 2.3

$$\hat{a}_k = \frac{2}{nw^*}\left[\sum_{i=1}^{n}\cos\left(\frac{k\pi x_i}{w^*}\right)\right] \qquad k = 1, 2, \ldots .$$

The terms $\sum_{i=1}^{40}\cos\left(\frac{k\pi x_i}{w^*}\right)$ for k = 1, 2, and 3 are 10.586, −2.888, and 1.332, respectively (Table 5). Then,

$$\hat{a}_1 = \frac{2}{40 \times 65}[10.586] = 0.008,$$

$$\hat{a}_2 = \frac{2}{40 \times 65}[-2.888] = -0.002, \text{ and}$$

$$\hat{a}_3 = \frac{2}{40 \times 65}[1.332] = 0.001.$$

The example above illustrates the basic calculations; substantial rounding error is introduced by doing the computations by hand (or on a small desk calculator) and in recording only a few significant digits (i.e., Table 5). The values we use for the first six $\hat{a}_k$'s were calculated on a computer, using program TRANSECT, and are $\hat{a}_1 = 0.008912$, $\hat{a}_2 = -0.002896$, $\hat{a}_3 = 0.001063$, $\hat{a}_4 = 0.001046$, $\hat{a}_5 = -0.001446$, and $\hat{a}_6 = 0.001480$.

Because the stopping rule requires knowledge of only w*, n, and the $\hat{a}_k$'s, one can determine the number of terms required to estimate f(0), and, subsequently, D. From Eq. (2.7)

$$\frac{1}{w^*}\left(\frac{2}{n+1}\right)^{1/2} \geq |\hat{a}_{m+1}|.$$

For the example, we want to choose a value of m such that

$$\frac{1}{65}\left(\frac{2}{41}\right)^{1/2} = 0.003398 \geq |a_{m+1}|.$$

We see that this inequality is satisfied for m = 1 because $0.003398 > |\hat{a}_{1+1}| = |\hat{a}_2| = 0.002896$. The stopping rule suggests that only one term in the Fourier series is needed to estimate f(0) in this example.

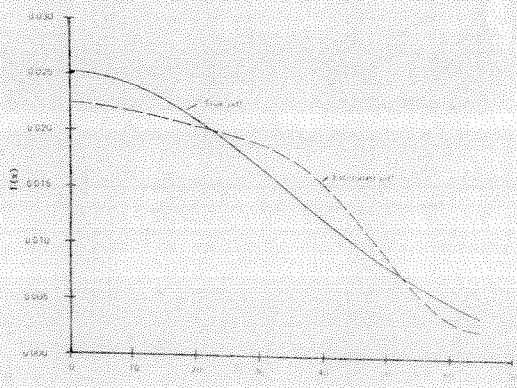


FIG. 27. Underlying half-normal density (σ=33.334, w=65, and f(0)=0.0252) with the Fourier series estimator of f(x) for a sample size n=40.

The estimate of $f(0)$ is made by using Eq. (2.2),

$$\hat{f}(0) = \frac{1}{w^*} + \sum_{k=1}^{m} \hat{a}_k,$$

and for this example,

$$\hat{f}(0) = \frac{1}{65} + \hat{a}_1$$
$$= 0.015385 + 0.008912$$
$$= 0.0242966 \text{ or } 0.0243.$$

Density can now be estimated by using Eq. (2.1)

$$\hat{D} = \frac{n\hat{f}(0)}{2L}$$
$$= \frac{40 \times 0.0243}{2 \times 1000}$$
$$= 0.000486 \text{ or } 4.86 \text{ objects/ha.}$$

This estimate compares fairly well with the true density of 5.04 objects/ha.

In this simple example, one needs only Eq. (2.4) to estimate the sampling variance of $\hat{a}_k$,

$$\hat{\text{var}}(\hat{a}_k) = \frac{1}{(n-1)}\left[\frac{1}{w^*}\left(\hat{a}_{2k} + \frac{2}{w^*}\right) - \hat{a}_k^2\right].$$

In the example, only the first coefficient is used to estimate $f(0)$ and D, therefore, only the variance of $\hat{a}_1$ is required (note, however, that $\hat{a}_2$ will be needed to compute this variance). In this case one needs only

$$\hat{\text{var}}(\hat{a}_1) = \frac{1}{39}\left[\frac{1}{65}\left(-0.002896 + \frac{2}{65}\right) - 0.008912^2\right]$$
$$= 0.000008959.$$

There are no covariances, therefore, Eq. (2.5) is not needed, because the variance–covariance matrix contains a single element, $\hat{\text{var}}(\hat{a}_1)$. Likewise, Eq. (2.6) is not needed, because the double sum of the variance–covariance matrix is merely $\hat{\text{var}}(\hat{a}_1)$ in this special case. Thus, from Eq. (2.6), the $\hat{\text{var}}(\hat{f}(0)) = \hat{\text{var}}(\hat{a}_1) = 0.000008959$. No actual area is involved because the data were simulated on a digital computer. It will be assumed, for illustration, that n is Poisson (as if the 40 objects were distributed randomly); therefore, $\hat{\text{var}}(n) = n$. Then,

$$\hat{\text{var}}(\hat{D}) = (\hat{D})^2\left[\frac{\hat{\text{var}}(n)}{n^2} + \frac{\hat{\text{var}}(\hat{f}(0))}{\hat{f}(0)^2}\right]$$
$$= (\hat{D})^2\left[\frac{40}{1600} + \frac{0.000008959}{(0.0243)^2}\right]$$
$$= (\hat{D})^2[0.025 + 0.015157]$$
$$= (0.000486)^2[0.040157]$$
$$= (0.00009739)^2.$$

The $\hat{\text{se}}(\hat{D}) = 0.00009739$ and $\hat{\text{cv}}(\hat{D}) = 0.20$ or a 20 percent coefficient of variation. The approximate 95 percent confidence interval based on $\hat{D} \pm 1.96\ \hat{\text{se}}(\hat{D})$ is (0.000293 to 0.000677).

To summarize, the FS estimate of density is 0.000486 ± 0.000191. Converting to objects per ha, $\hat{D} = 4.86 \pm 1.91$. In that example, the method performed fairly well; the estimated percentage relative bias (PRB)$= 100 \times (\hat{D} - D)/D = -3.6$.

As an extended example, now consider the case where $m = 3$. That case is included to illustrate better the estimation of the sampling variance of $\hat{f}(0)$ and $\hat{D}$. The estimation of the a_k's (e.g., Table 5) has already been shown (i.e., $\hat{a}_1 = 0.008912$, $\hat{a}_2 = -0.002896$, $\hat{a}_3 = 0.001063$, $\hat{a}_4 = 0.001046$, $\hat{a}_5 = -0.001446$, and $\hat{a}_6 = 0.001480$). Given that $m = 3$, the estimate of $f(0)$ is computed from Eq. (2.2),

$$\hat{f}(0) = \frac{1}{w^*} + \sum_{k=1}^{m} \hat{a}_k$$
$$= \frac{1}{65} + (\hat{a}_1 + \hat{a}_2 + \hat{a}_3)$$
$$= 0.022463.$$

Density is estimated easily from Eq. (2.1),

$$\hat{D} = \frac{n\hat{f}(0)}{2L}$$
$$= \frac{40 \times 0.022463}{2 \times 1000}$$
$$= 0.000449 \text{ or } 4.49 \text{ objects/ha.}$$

This estimate is inferior to the estimates

of 4.86 obtained by using the FS approach and the stopping rule (which suggested m = 1), and it has a larger sampling variance because 3 cosine terms are used.

The elements of the variance–covariance matrix of the coefficients $\hat{a}_k$ are computed by using Eq. (2.4) and (2.5); the matrix appears below (note that the matrix is symmetric about the diagonal):

$$\begin{bmatrix} 0.000008959 & 0.000004597 & -0.000000973 \\ 0.000004597 & 0.000012335 & 0.000003024 \\ -0.000000973 & 0.000003024 & 0.00001693 \end{bmatrix}.$$

Although m = 3, one needs 2m = 6 coefficients to estimate the variance–covariance matrix.

The sampling variance of $\hat{f}(0)$ is computed by merely summing all the elements in the variance–covariance matrix. Summing the 9 elements, $\hat{\text{var}}(\hat{f}(0)) = 0.000047296$.

Finally, the sampling variance of the density estimate can be computed. We illustrate the computation by using the Poisson assumption (because no actual area is involved). Using that assumption as an illustration, $\hat{\text{var}}(n) = n$. Then,

$$\begin{aligned}
\hat{\text{var}}(\hat{D}) &= \hat{D}^2\left[\frac{\hat{\text{var}}(n)}{n^2} + \frac{\hat{\text{var}}(\hat{f}(0))}{(\hat{f}(0))^2}\right] \\
&= \hat{D}^2\left[\frac{1}{n} + \frac{\hat{\text{var}}(\hat{f}(0))}{(\hat{f}(0))^2}\right] \\
&= (0.000449)^2\left[\frac{1}{40} + \frac{0.0000473}{(0.022463)^2}\right] \\
&= 0.0000000239 \\
\hat{\text{se}}(\hat{D}) &= \left(\hat{\text{var}}(\hat{D})\right)^{1/2} = 0.0001547.
\end{aligned}$$

An approximate 95 percent confidence interval can be computed as $\hat{D} \pm 1.96\ \hat{\text{se}}(\hat{D})$, which is 1.46 to 7.52 objects/ha in the example. One might expect the actual confidence interval coverage to be less than 95 percent because the estimator of f(0) is based on a trade-off between variance and bias. The trade-off is accomplished by the stopping rule based on the mean integrated squared error, where MISE = variance + bias^2. The actual confidence interval coverage may be less than 95 percent because the bias need not be zero and the "best" FS model is not always selected by the stopping rule. The extent that those intervals are approximate is not known and is the subject of needed research. Some theoretical discussion of this subject is given in PART 3.

The estimated PRB for this example is −10.9 (Fig. 27). The average PRB for 25 replications of this experiment (with m fixed at 3) was 4.0. This seems quite good considering the small sample size of n = 40. Further properties of the FS approach are discussed in PART 3.

Stake Data

Laake (1979, unpublished master's thesis, Utah State University, Logan, Utah) conducted an investigation of line transect sampling by placing a known number of wooden stakes in a sagebrush meadow east of Logan, Utah. Lines were walked by graduate students in the Department of Wildlife Science at Utah State University. The sample data on perpendicular distances from one such survey are used as a second example. The observer found 68 stakes within a fixed width of 20 m (w = 20 m) on a single line transect 1,000 m long (L = 1,000). The actual density was known to be 0.00375, or 37.5 stakes/ha. The true form of f(x), the pdf, was unknown and varied from observer to observer. The perpendicular distances x_i for the 68 stakes found are shown in Table 6. Note that the greatest distance was 18.60 m (that is, the largest order statistic $X_{(68)} = 18.60$), and that value was used for w* (i.e., $w^* = 18.60 < w = 20.0$). In many situations, one may wish to select a value for w* such that 1–3 percent of the outliers are excluded from the analysis (see *Example 6 Data Truncation* and Fig. 44). However, for the purpose of this example, w* = 18.60 will suffice.

Estimates of $\hat{a}_k$ for k = 1, 2, . . . are computed from Eq. (2.3),

TABLE 6.—EXAMPLE OF LINE TRANSECT DATA FROM A SAMPLE OF WOODEN STAKES OF KNOWN DENSITY; n = 68, w^* = 18.60 m, w = 20 m, L = 1,000 m, AND D = 0.00375 STAKES PER METER. THE DATA SHOWN ARE THE PERPENDICULAR DISTANCES IN METERS

1.	2.02	35.	3.79
2.	0.45	36.	15.24
3.	10.40	37.	3.47
4.	3.61	38.	3.05
5.	0.92	39.	7.93
6.	1.00	40.	18.15
7.	3.40	41.	10.05
8.	2.90	42.	1.11
9.	8.16	43.	1.27
10.	6.47	44.	13.72
11.	5.66	45.	6.25
12.	2.95	46.	3.59
13.	3.96	47.	9.04
14.	0.09	48.	7.68
15.	11.82	49.	4.89
16.	14.23	50.	9.10
17.	2.44	51.	3.25
18.	1.61	52.	8.49
19.	31.31	53.	6.08
20.	6.50	54.	0.40
21.	8.27	55.	9.33
22.	4.85	56.	0.53
23.	1.47	57.	1.23
24.	18.60	58.	1.67
25.	0.41	59.	4.53
26.	0.40	60.	3.12
27.	0.20	61.	3.05
28.	11.59	62.	6.60
29.	3.17	63.	4.40
30.	7.10	64.	4.97
31.	10.71	65.	3.17
32.	3.86	66.	7.67
33.	6.05	67.	18.16
34.	6.42	68.	4.08

$$\hat{a}_k = \frac{2}{nw^*}\left|\sum_{i=1}^{n}\cos\left(\frac{k\pi x_i}{w^*}\right)\right|, \qquad k = 1, 2, \ldots .$$

In addition, $a_0 \equiv 2/w^*$. After careful computation, we have $a_0 = 0.107527$, $\hat{a}_1 = 0.050130$, $\hat{a}_2 = 0.010910$, $\hat{a}_3 = -0.004979$, and $\hat{a}_4 = -0.000325$.

Use of the stopping rule (Eq. (2.7)) enables an objective choice of m, the number of FS terms in the estimator of f(0). The stopping rule indicates that 2 terms are appropriate;

$$\frac{1}{w^*}\left(\frac{2}{n+1}\right)^{1/2} \geq |a_{m+1}|$$

or

$$\frac{1}{18.60}\left(\frac{2}{69}\right)^{1/2} = 0.00915 \geq |a_{2+1}| = 0.00498.$$

Therefore, the estimate of f(0) is computed by using Eq. (2.2),

$$\hat{f}(0) = \frac{1}{w^*} + \sum_{k=1}^{m}\hat{a}_k,$$

and for that example,

$$\begin{aligned} f(0) &= \frac{1}{18.60} + \sum_{k=1}^{2}\hat{a}_k \\ &= 0.053763 \\ &\quad + (0.050130 + 0.010910) \\ &= 0.114803. \end{aligned}$$

The estimate of density is made by using Eq. (2.1)

$$\hat{D} = \frac{n\hat{f}(0)}{2L} = 0.0039033 \text{ or } 39.0 \text{ stakes per ha.}$$

In that example, the estimated PRB is 4.

The sampling variance of $\hat{D}$ requires the use of Eq. (2.4)–(2.6). The variance of $\hat{a}_1$ and $\hat{a}_2$ is computed as

$$\begin{aligned} \hat{\text{var}}(\hat{a}_1) &= \frac{1}{67}\left[\frac{1}{18.6}(\hat{a}_2 + a_0) - (\hat{a}_1)^2\right] \\ &= 0.0000575 \end{aligned}$$

$$\begin{aligned} \hat{\text{var}}(\hat{a}_2) &= \frac{1}{67}\left[\frac{1}{18.6}\left(\hat{a}_4 + \frac{2}{18.6}\right) - (\hat{a}_2)^2\right] \\ &= .0000842 \end{aligned}$$

and the covariance between $\hat{a}_1$ and $\hat{a}_2$ is

$$\begin{aligned} \hat{\text{cov}}(\hat{a}_1,\hat{a}_2) &= \frac{1}{(n-1)}\left[\frac{1}{w^*}(\hat{a}_{1+2} + \hat{a}_{2-1}) - (\hat{a}_1\hat{a}_2)\right] \\ &= \frac{1}{67}\left[\frac{1}{18.6}(\hat{a}_3 + \hat{a}_1) - (\hat{a}_1\hat{a}_2)\right] \\ &= 0.0000281. \end{aligned}$$

Because the stopping rule indicated m = 2, a 2 by 2 variance–covariance matrix with elements is

$$\begin{bmatrix} 0.0000575 & 0.0000281 \\ 0.0000281 & 0.0000842 \end{bmatrix}.$$

Summation of the 4 elements in the variance–covariance matrix (Eq. 2.6) gives $\hat{\text{var}}(\hat{f}(0)) = 0.0001979$.

Several approaches can be taken to compute the estimate of the sampling variance of $\hat{D}$. The stakes were placed randomly, therefore, n is distributed as a binomial with parameters N and p, where N is known (N = 150 in the data). The estimator for p is $\hat{p} = n/N$. If p is small, the Poisson approximation to the binomial is good. However, $\hat{p} = 68/150 = 0.4533$ is large, so the Poisson approximation can be expected to be poor. Here, for illustration, we explore both approaches.

If the Poisson assumption holds, the $\hat{\text{var}}(n)$ is simply

$$\hat{\text{var}}(n) = n.$$

Assuming n is binomial, the $\hat{\text{var}}(n)$ is

$$\hat{\text{var}}(n) = N\hat{p}(1 - \hat{p})$$

and because $\hat{p} = n/N$,

$$\hat{\text{var}}(n) = N\left(\frac{n}{N}\right)\left(1 - \frac{n}{N}\right)$$

$$= n\left(1 - \frac{n}{N}\right).$$

The general estimator for the sampling variance of $\hat{D}$ is appropriate under either assumption.

$$\hat{\text{var}}(\hat{D}) = (\hat{D})^2\left[\frac{\hat{\text{var}}(n)}{(n)^2} + \frac{\hat{\text{var}}(\hat{f}(0))}{(\hat{f}(0))^2}\right].$$

Concerns about Poisson or binomial assumptions for n involve only the first term in the brackets. Note, however, under the binomial assumption

$$\frac{\hat{\text{var}}(n)}{(n)^2} = \frac{1}{n}\left(1 - \frac{n}{N}\right),$$

which shows that as N gets large the Poisson assumption holds, i.e.,

$$\frac{\hat{\text{var}}(n)}{(n)^2} = \frac{1}{n}.$$

One computes the estimate of $\text{var}(\hat{D})$, using the Poisson assumption

$$\hat{\text{var}}(\hat{D}) = (\hat{D})^2\left[\frac{n}{n^2} + \frac{\hat{\text{var}}(\hat{f}(0))}{(\hat{f}(0))^2}\right]$$

$$= (\hat{D})^2\left[\frac{1}{68} + \frac{0.0001979}{(0.114803)^2}\right]$$

$$= 0.0000004528$$

$$\hat{\text{se}}(\hat{D}) = 0.000673.$$

The estimate of $\text{var}(\hat{D})$ under the binomial assumption is slightly smaller, even though the Poisson assumption is poor because $\hat{p} = 0.45$

$$\hat{\text{var}}(\hat{D}) = (\hat{D})^2\left[\frac{1}{n}\left(1 - \frac{n}{N}\right) + \frac{\hat{\text{var}}(\hat{f}(0))}{(\hat{f}(0))^2}\right]$$

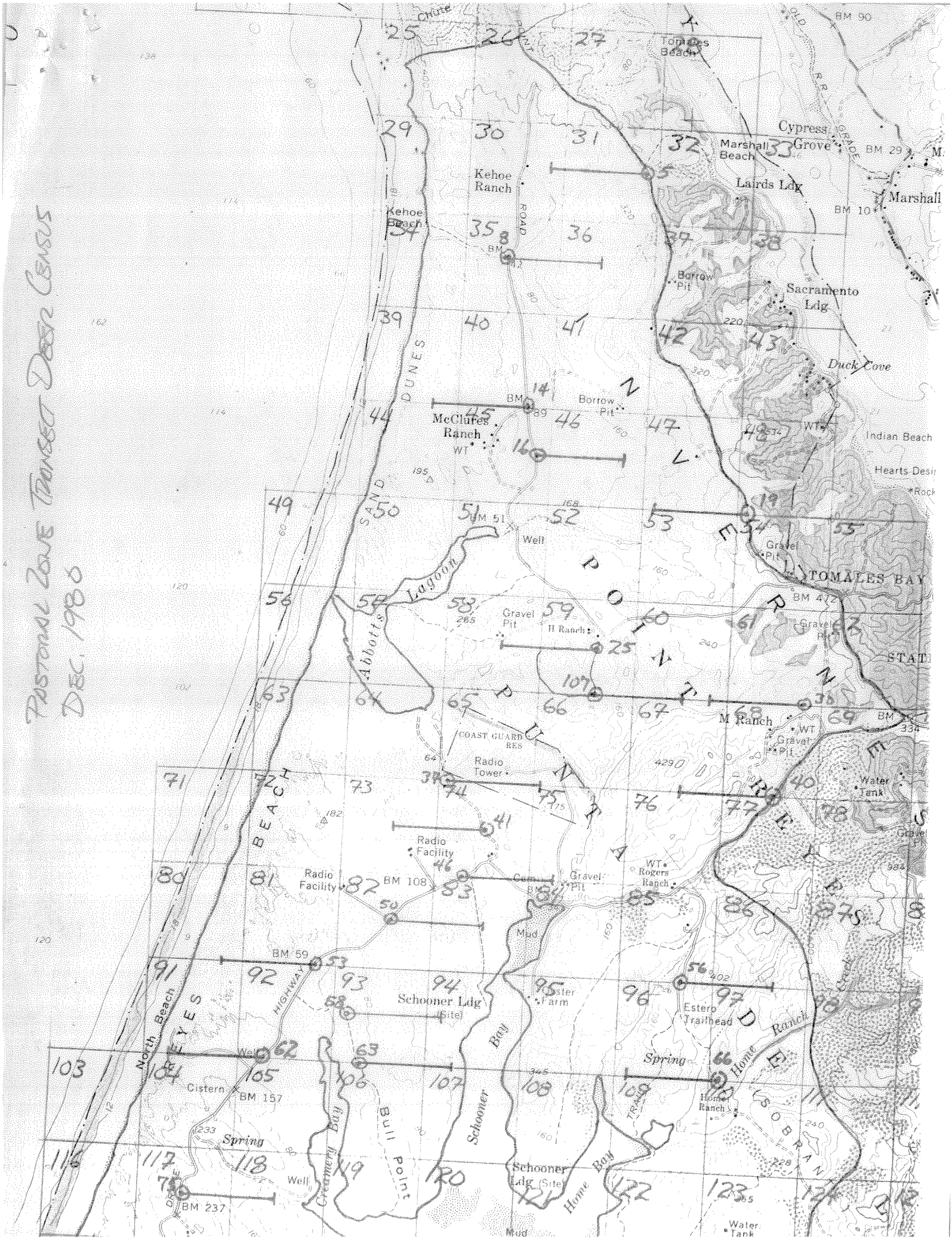
$$= (\hat{D})^2[0.00804 + 0.0150155]$$

$$= 0.0000003513$$

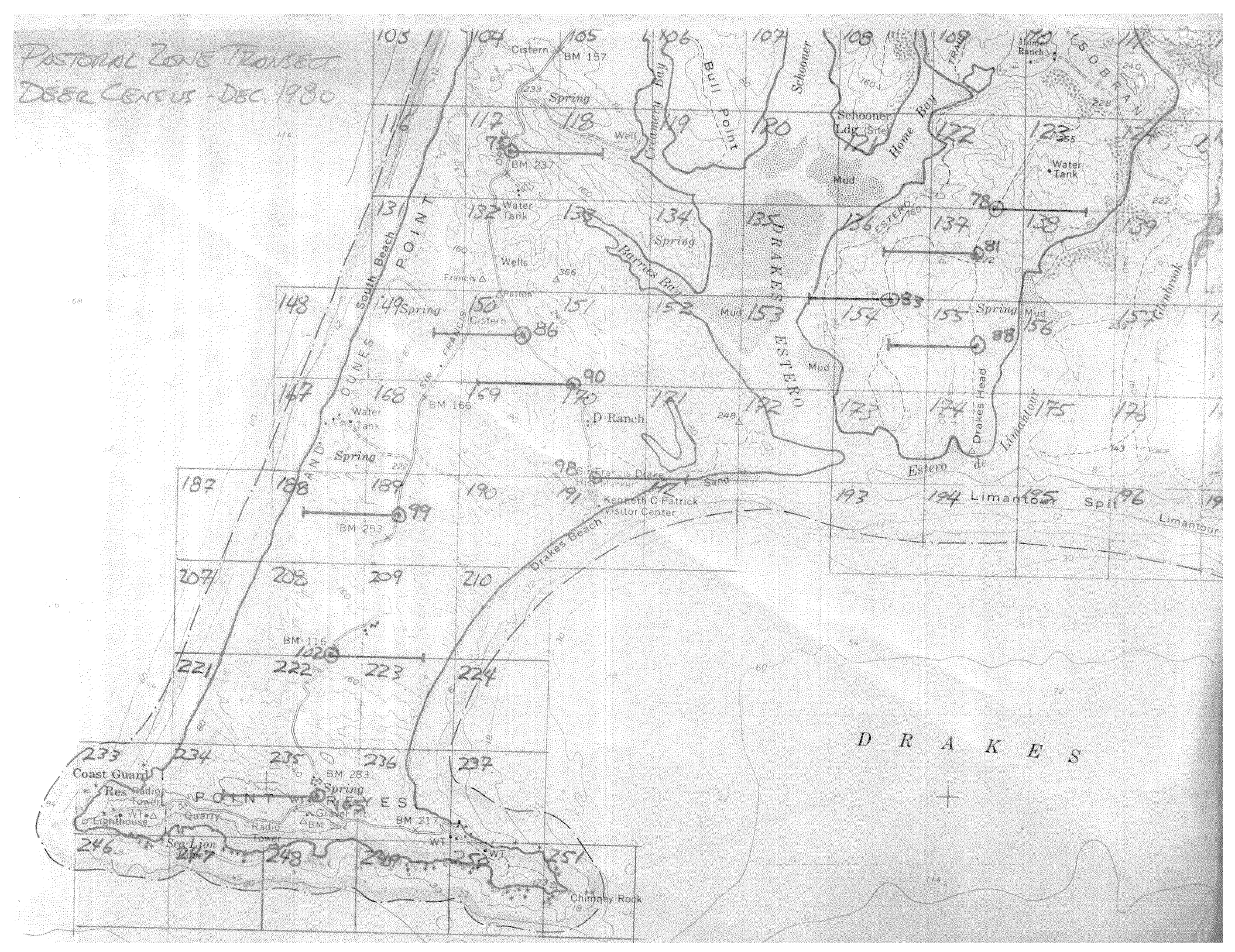
$$\hat{\text{se}}(\hat{D}) = 0.000593.$$

Hemingway's Data

P. Hemingway (pers. comm.) studied various ungulates in Africa using line transect sampling. Here, some of his 1971 data are used to illustrate use of the FS procedure on ungrouped perpendicular distance data. The emphasis of this example is on the interpretation of the computer output from program TRANSECT. During the survey, 73 animals were detected on a 60-km transect. Only the sighting distance (SD) and the measured sighting angle (MESANG) were recorded in the field; program TRANSECT computes the perpendicular distance (PD) from them. A partial listing of the data and various summary statistics are given in Fig. 28a. (Program TRANSECT lists all the data; space limitations prevent displaying the entire list here.) For example, the average perpendicular distance was 99.25 m, with a standard deviation of 82.37. Finally, a matrix of correlation coefficients and significance levels is printed. The perpendicular distance data are printed, in increasing order (Fig. 28b). Such ordered data are called "order statistics" and are denoted as $X_{(i)}$ for i = 1,



Pastoral Love Thwarted Deer Census
Dec 1980


PASTORAL ZONE TRANSECT
DEER CENSUS - DEC. 1980
Cistern
BM 157
Spring
Well
Creamery Bay
Bull Point
Schooner
Schooner Ldg (Site)
Home Bay
TRAIL
Historic Ranch
SO BRAN
Water Tank
Mud
BM 237
Water Tank
Wells
Francis
Patton
Spring
Barries Bldg
DRAKES ESTERO
ESTERO
Glenbrook
Spring
Cistern
Mud
SOUTH BEACH
POINT
SIR FRANCIS
BM 166
D Ranch
Water Tank
Spring
Sir Francis Drake Historical Marker
Sand
Kenneth C Patrick Visitor Center
Drakes Beach
Drakes Head
Limantour
Estero de
Limantour Spit
BM 253
BM 116
Coast Guard
Res
Radio Tower
Lighthouse
Quarry
POINT REYES
BM 283
Spring
Gravel Pit
BM 352
BM 217
Sea Lion
Chimney Rock
DRAKES